Improving Uplink Adaptive Antenna Algorithms for WCDMA by Covariance Matrix Compensation

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***Abstract*—This document addresses the estimation of long-term spatial covariance matrices. Many beamforming techniques are based on those matrices. In the past, they often have been assumed to be perfectly known, since long averaging processes can be applied. However, a systematic error is committed when using conventional methods for the estimation.**
The occurrence and the performance impact of this error is discussed in this paper. A simple compensation scheme is introduced which completely removes this error. Simulation results are presented for a multi-user WCDMA environment. For beamforming methods based on maximizing the signal power, gains of more than 2dB are observed. The impact of the systematic error on SINR maximizing algorithms is minor so that the improvement of the compensation scheme is slight.
The results show, that an estimate for the interference covariance is needed in all cases, either for compensating the signal covariance or for SINR based techniques.

I. Introduction

ADAPTIVE antenna techniques are among the most promising means to enhance capacity in mobile communication systems. A large number of algorithms has been proposed. With the wide-sense-stationarity (WSS) assumption [1] the spatial structure of the mobile channel can be considered constant over a wide range [2] in spite of a moving mobile station. Therefore, most of the algorithms [3] are based on spatial covariance matrices [4] expressing this spatial structure. The reliability for estimating these matrices increases due to long averaging.

For this reason, long-term spatial covariance matrices have been assumed perfectly known in most of the publications. However, even with perfect ergodicity and infinite averaging, a systematic error remains.

This paper addresses this error. Its occurrence as well as its impact on the performance is described for a wideband code division multiple access (WCDMA) uplink regime. A simple compensation scheme is presented which removes the error and almost fully returns the degradations caused by the real estimation of the covariance matrices.

Section II introduces the system model and briefly reviews the antenna algorithms which are considered in the subsequent sections, namely the fixed beamformer, the signal based eigen beamformer and the signal and interference based eigen beamformer [5][6][7]. For the sake of convenience, a frequency flat channel is assumed in this first part. Furthermore, the beam SINR (signal-to-noise-plus-interference ratio) is defined. The typical procedure for estimating the signal covariance matrix and the noise covariance matrix is given in Section III. The covariance compensation scheme is presented in Section IV. In addition, the impact on the SINR is discussed for the three antenna strategies and the extension to frequency selective channels is described. Finally, simulation results for an UTRA FDD[1] uplink scenario with multiple users are shown in Section V. Section VI concludes this work.

II. Smart Antenna Concepts for WCDMA

In WCDMA systems, the number of quasi-orthogonal users interfering with each other usually is much larger than the number of antenna elements. Hence, interferer nulling which often requires precise direction-finding is not the primary purpose of WCDMA antenna algorithms. Instead, the signal and interference characteristics is compiled into long-term spatial covariance matrices. In this section, after introducing the system model we will describe antenna concepts that are shown to match the WCDMA philosophy. These concepts will be used to discuss the covariance compensation later on.

A. System Model

Figure 1 depicts the system model used throughout this work. A transmitted scalar user signal $s(t)$ is received by K_a antenna elements. We assume the propagation channel to be frequency-flat, so that it is expressed by the multiplicative coefficient vector $\vec{\alpha} \in \mathcal{C}^{K_a \times 1}$. We will extend to the frequency-selective case in Section IV-D.

A vector noise process $\vec{n}(t) \in \mathcal{C}^{K_a \times 1}$ superimposes to the

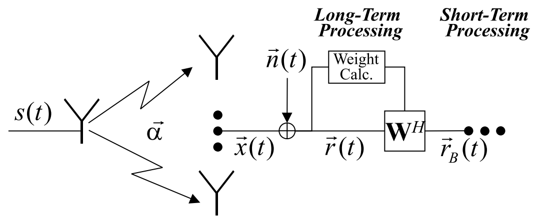

Fig. 1. System Model.

received signal of the desired user $\vec{x}(t) \in \mathcal{C}^{K_a \times 1}$. The noise comprises thermal noise as well as intercell and intracell interference. Hence, the total received signal $\vec{r}(t) \in \mathcal{C}^{K_a \times 1}$ writes as

$$\vec{r}(t) = \vec{\alpha} \cdot s(t) + \vec{n}(t) = \vec{x}(t) + \vec{n}(t). \quad (1)$$

For the sake of convenience, we will omit the parentheses (t) in the sequel. A set of K_b beamforming weights $\vec{w}_k^H \in \mathcal{C}^{1 \times K_a}$ with $1 \leq k \leq K_b$ is calculated based on the spatial long-term properties of the channel. Possible methods for the weight

[1] UMTS Terrestrial Radio Access, Frequency Division Duplex mode

calculation are e.g. direction finding techniques or those described in the subsequent sections.
The vectors $\vec{w}_k$ are summarized in the matrix $(\vec{w}_1, \vec{w}_2, ..., \vec{w}_{K_b}) = \mathbf{W} \in \mathcal{C}^{K_a \times K_b}$. This matrix transforms the K_a received signals $\vec{r}$ into K_b beam signal $\vec{r}_B = \mathbf{W}^H \cdot \vec{r}$ with $\vec{r}_B \in \mathcal{C}^{K_b \times 1}$, which then are fed into the short-term processing (e.g. maximum ratio combining).
A lot of beamforming techniques are based on the spatial covariance matrix of the desired signal and of the noise (including the interference). The signal covariance matrix is [4]

$$\mathbf{R}_{xx} = \mathbf{E}\left\{\vec{x} \cdot \vec{x}^H\right\} = \mathbf{E}\left\{\vec{\alpha} \cdot \vec{\alpha}^H\right\} \tag{2}$$

where without the loss of generality $\mathbf{E}\left\{|s|^2\right\} = 1$ was assumed. The interference and noise covariance matrix writes as

$$\mathbf{R}_{nn} = \mathbf{E}\left\{\vec{n} \cdot \vec{n}^H\right\}. \tag{3}$$

Using these conventions we will review three different techniques to determine the beamforming weights $\mathbf{W}^H$ in the sequel.

B. Fixed Beamformer

The fixed beamformer (FBF) [5] selects the best beams from a predefined set. Figure 2 shows an example, where a uniform linear array (ULA) with $K_a = 6$ elements and $\lambda/2$ spacing is used to form 6 uniformly spaced beams within a 120^o sector.
The decision which beams to select is made on the average

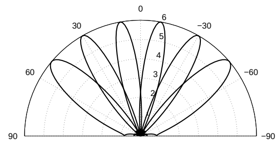

Fig. 2. Set of Fixed Beams.

signal power $P_{F,i}$ collected by the ith beam. By help of the signal covariance matrix $\mathbf{R}_{xx}$, the ith beam power is expressed as

$$P_{F,i} = \vec{w}_{F,i}^H \cdot \mathbf{R}_{xx} \cdot \vec{w}_{F,i} \tag{4}$$

where $\vec{w}_{F,i}^H$ is the weight vector generating the ith beam.

C. Signal based Eigen Beamformer

Similar to the fixed beamformer, the signal based eigen beamformer (SB EBF) [6][7] also maximizes the average power collected by a generated beam, but it does not make any restrictions on the beam weights. The optimization problem

$$P(\vec{w}) = \vec{w}^H \cdot \mathbf{R}_{xx} \cdot \vec{w} \tag{5}$$

is solved by an eigenvalue decomposition of $\mathbf{R}_{xx}$:

$$\mathbf{R}_{xx} \cdot \vec{w} = \lambda \cdot \vec{w}. \tag{6}$$

This yield K_a orthogonal eigen vectors. Each of them generates a beam with the corresponding eigenvalues representing the average collected power.

D. Signal and Interference based Eigen Beamformer

Instead of maximizing the signal power, the signal and interference based eigen beamformer (SIB EBF) [6][7] maximizes the signal-to-noise-and-interference-ratio (SINR). The SINR $\gamma(\vec{w}^H)$ on a beam $\vec{w}^H$ writes as

$$\gamma(\vec{w}^H) = \frac{\vec{w}^H \cdot \mathbf{R}_{xx} \cdot \vec{w}}{\vec{w}^H \cdot \mathbf{R}_{nn} \cdot \vec{w}} \tag{7}$$

and is maximized by the generalized eigenvalue decomposition (cf. rayleigh quotient)

$$\mathbf{R}_{xx} \cdot \vec{w} = \lambda \cdot \mathbf{R}_{nn} \cdot \vec{w}. \tag{8}$$

Again we end up in K_a orthogonal beams, where the eigenvalues here account for the corresponding SINR.

E. Beam Selection and Beam SINR

In all concepts, the best beams among the generated beam candidates in terms of beam power (FBF and SB EBF) or SINR (SIB EBF) are selected. This is typically done by applying a threshold to the decision variable. For WCDMA usually values between 6dB and 10dB below the maximum are chosen. Beams below this threshold do not carry significant signal components and are therefore discarded [8].
Finally, a low number of beams $\vec{w}_i^H$ survives. The actual SINR on beam i is

$$\gamma_i = \frac{\vec{w}_i^H \cdot \mathbf{R}_{xx} \cdot \vec{w}_i}{\vec{w}_i^H \cdot \mathbf{R}_{nn} \cdot \vec{w}_i}. \tag{9}$$

In a receiver unit, the beam signals are e.g. maximum-ratio combined to track the short-term fading effects of the channel. We will use the beam SINR to asses the impact of the compensation scheme in Section IV-B.

III. Estimation of Covariance Matrices

In this section we will describe how to obtain estimates of the covariance matrices $\mathbf{R}_{xx}$ and $\mathbf{R}_{nn}$ as defined in (2) and in (3). We assume, that we are able to carry out the expectation operations by averaging over the short-term fading effects. This is equivalent to perfect ergodicity (cf. WSS [1]) and infinite averaging over time.

A. Signal Covariance Matrix

In mobile communication systems with coherent detection we usually have a training sequence available. This allows to get estimates of the channel coefficients $\vec{\alpha}$ e.g. by correlating the received signal with the training sequence

$$\hat{\vec{\alpha}} = \sqrt{\frac{1+\beta^2}{\beta^2}} \cdot \frac{1}{L} \cdot \sum_{i=1}^{L} \vec{r}_i \cdot p_i \tag{10}$$

where $\vec{r}_i$ and p_i are the ith sample of the received signal and the training sequence, respectively, L is the length of the training sequence and β^2 is the power ratio of the samples of the training sequence to the samples of the transmitted data samples.
Replacing the coefficient vector $\vec{\alpha}$ in (2) by its estimate $\hat{\vec{\alpha}}$ yields

$$\hat{\mathbf{R}}_{xx} = \mathbf{E}\left\{\hat{\vec{\alpha}} \cdot \hat{\vec{\alpha}}^H\right\} = \mathbf{R}_{xx} + \frac{1+\beta^2}{\beta^2 \cdot L} \cdot \mathbf{R}_{nn}. \tag{11}$$

That is, even with perfect long-term averaging, a noise component remains which is obviously reduced with increasing training length L and with increasing fractional training power β^2. However, in usual WCDMA operational areas, both signal and noise component of the estimate might have the same order of magnitude.

B. *Noise Covariance Matrix*

Typical for WCDMA, any interference and noise measurement is derived from the total received signal $\vec{r}$. Its spatial covariance matrix is

$$\mathbf{R}_{rr} = \mathbf{E}\left\{\vec{r} \cdot \vec{r}^H\right\} = \mathbf{R}_{xx} + \mathbf{R}_{nn}. \qquad (12)$$

In a highly loaded WCDMA system, the spread signal component is very weak compared with thermal noise, intracell- and intercell-interference. Hence, the approximation $\mathbf{R}_{rr} \approx \mathbf{R}_{nn}$ is applied.

IV. Compensation Scheme

A. *Covariance Compensation*

Having a closer look on the perturbed estimates $\hat{\mathbf{R}}_{xx}$ and $\mathbf{R}_{rr}$ in (11) and (12), we simply get the correct matrices $\mathbf{R}_{xx}$ and $\mathbf{R}_{nn}$ by solving a linear system of equations:

$$\mathbf{R}_{xx} = \frac{\beta^2 \cdot L}{(L-1)\cdot\beta^2 - 1} \cdot \left(\hat{\mathbf{R}}_{xx} - \frac{1+\beta^2}{\beta^2 \cdot L} \cdot \mathbf{R}_{rr}\right) \qquad (13)$$

$$\mathbf{R}_{nn} = \frac{\beta^2 \cdot L}{(L-1)\cdot\beta^2 - 1} \cdot \left(\mathbf{R}_{rr} - \hat{\mathbf{R}}_{xx}\right). \qquad (14)$$

That is, we can completely cancel the noise component in $\hat{\mathbf{R}}_{xx}$ and the signal component in $\mathbf{R}_{rr}$ without any approximation.
In the next section, we will study the systematic error committed when using $\hat{\mathbf{R}}_{xx}$ and $\mathbf{R}_{rr}$ instead of the correct matrices $\mathbf{R}_{xx}$ and $\mathbf{R}_{nn}$ using parameters typical for WCDMA.

B. *Impact on SINR performance*

According to the UTRA FDD AMR12.2 [9] service we assume $\beta^2 = -2.69$dB and $L = 1536$ in this section. The array is a uniform linear array (ULA) with $K_a = 6$ elements and $\lambda/2$ spacing. The carrier-to-interference-ratio (C/I) is set to $C/I = -26$dB. $\mathbf{R}_{xx}$ is determined by a user with direction of arrival (DOA) of 30^o and an angular spread of 10^o.
To clarify the influence of the compensation we consider two extreme cases for the structure of $\mathbf{R}_{nn}$. In the first case (*white interference*), we have uncorrelated noise only, i.e. $\mathbf{R}_{nn}$ is a scaled identity. In the second case (*coloured interference*), $\mathbf{R}_{nn}$ is dominated by a strong interferer with DOA of -30^o and an angular spread of 10^o. 5% uncorrelated noise is added, in order to avoid numerical problems due to a singular matrix $\mathbf{R}_{nn}$.
For the three beamforming concepts, the following figures depict the decision variable normalized to its strongest value in the left column, and the actual SINR (9) of the beam candidates. Note, that the order of the beams is not the same in the left and right column, but in the the right column the beams are sorted with respect to the decision variable. The true values and the uncompensated results are shown for the two interference cases.

Figure 3 shows the result for the fixed beamformer, where the beam set of Figure 2 is used. In the case of white interference, the strongest beam signal is generated by the beam pointing into the direction of the desired user. However, all other uncompensated values of the decision variable pretend significant signal power as well. Depending on the height of an applied threshold, we might select some noise beams in addition to the signal carrying beam, which degrades the performance.
With a dominant interferer, the remaining interference com-

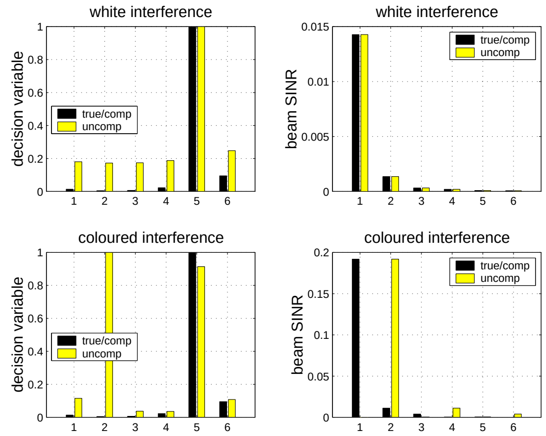

Fig. 3. Fixed Beamformer.

ponent in the decision variable is obviously even stronger than the desired signal component. Hence, the uncompensated case decides the interference beam to be the strongest one, although it hardly carries any signal power.
While the beams in the fixed beamformer are predefined, the eigen beamformer concept adaptively creates the beams. With white interference, the generated beams obviously are the same in the compensated case and in the uncompensated case, since we observe same beam SINRs in Figure 4. Adding an identity to a matrix does not change its eigenvectors, but offsets its eigenvalues. Hence, similar to the fixed beamformer, noise dimensions might be selected, but the beams and their order with respect to the decision variable are correct.
With coloured interference, the generated beams do not match

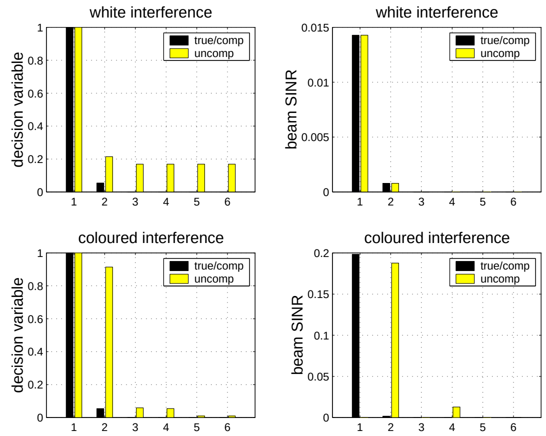

Fig. 4. Signal Based Eigenbeamformer.

the correct beams. The decision variable pretends two significant beams, whereas the beam supposed to be the strongest again hardly carries any signal component.

As described in Section II-C with the signal and interference based eigen beamformer the beam generation and selection is based on the SINR. Having a closer look on the eigenvalue problem (8) it is clear that mutual perturbation of the two matrices does not affect the eigenvectors, but transforms the eigenvalues. Hence, using the estimates $\hat{\mathbf{R}}_{xx}$ and $\mathbf{R}_{rr}$ generates the correct beams and the corresponding decision variable $\hat{\lambda}$

$$\hat{\lambda} = \frac{\lambda + c}{\lambda + 1} \quad \text{with} \quad c = \frac{1+\beta^2}{\beta^2 \cdot L}. \tag{15}$$

We observe three important properties, which are reflected by the results in Figure 5:

1) The function is monotone as long as $c < 1$, which usually is the case in mobile communication. That is, the order of the eigenvalues is not changed by the transform.
2) The function is convex, i.e. the range of the transformed eigenvalues is smaller than of the original ones. With non-perfect long-term averaging, this leads to more decision errors due to the variance of the long-term parameters.
3) The function exhibits the offset $\lambda = 0 \rightarrow \hat{\lambda} = c$. As in the previous concepts this possibly forces the beam selector to choose more beams than necessary, especially in very low λ-regions, i.e. very low C/I ranges as in WCDMA.

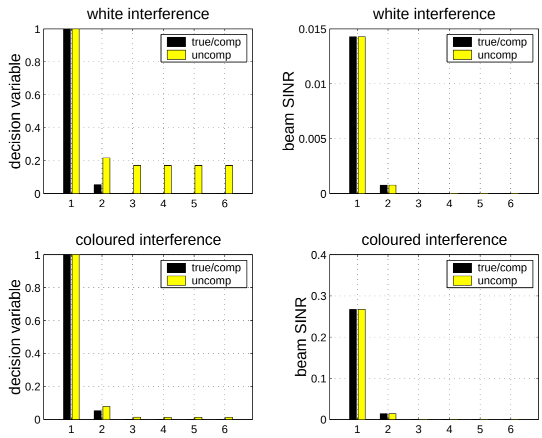

Fig. 5. Signal and Interference Based Eigenbeamformer.

Another consequence of (15) is, that instead of compensating for the covariances as in (13) and (14) we can equivalently compensate for the eigenvalues using

$$\lambda = \frac{\hat{\lambda} - c}{1 - \hat{\lambda}}. \tag{16}$$

Similar simplifications are possible for the other schemes, which saves a lot of complexity. This is straight forward and is not elaborated here. If we also based the fixed beamformer on the SINR instead of the signal power the simplified compensation exactly would look like (16) with λ being replaced by the beam SINR.

C. Interpretation

Assessing the results of the previous section, we expect high performance degradations for the signal power based techniques (FBF and SB EBF), since a large noise/interference component remains in the signal covariance matrix and in the decision variable. This forces wrong beams to be generated (SB EBF) or wrong beams to be selected (FBF).

The impact of the systematic error on SINR based methods (SIB EBF) seems to be less dramatic, as the noise/interference component appears in the denominator as well. Here, the only effect is that the range of the decision variable is offset and compressed which slightly increases sensibility against other estimation errors such as non-perfect long-term averaging.

Before we demonstrates these insights by simulation results, we extend the compensation scheme to frequency selective channels.

D. Compensation Scheme for frequency-selective Channels

In typical mobile communication systems, the energy occurs concentrated within the channel impulse responses. In WCDMA, each concentration (*delay tap*) usually is considered as a flat channel, where the other concentrations act as temporally uncorrelated noise. The signal paths then are e.g. maximum-ratio- combined [8].

In [6] the authors proposed to set up separate covariances $\mathbf{R}_{xx}^{(i)}$ and $\mathbf{R}_{nn}^{(i)}$ for each delay tap i and to execute a tapwise beamforming. Note that the noise covariance slightly differs over the different taps. Equivalent to the derivation in Section III, we get the estimates $\hat{\mathbf{R}}_{xx}^{(i)}$ and $\mathbf{R}_{rr}$ and according to (13) and (14) the compensation scheme becomes

$$\mathbf{R}_{xx}^{(i)} = \frac{\beta^2 \cdot L}{(L-1)\cdot\beta^2 - 1} \cdot \left(\hat{\mathbf{R}}_{xx}^{(i)} - \frac{1+\beta^2}{\beta^2 \cdot L} \cdot \mathbf{R}_{rr} \right) \tag{17}$$

$$\mathbf{R}_{nn}^{(i)} = \frac{\beta^2 \cdot L}{(L-1)\cdot\beta^2 - 1} \cdot \left(\mathbf{R}_{rr} - \hat{\mathbf{R}}_{xx}^{(i)} \right). \tag{18}$$

As in the flat case, this again can be translated to a simpler compensation for e.g. the tapwise eigenvalues (cf. (16)).

In the following section, we will apply the compensation to a multiple user UTRA FDD environment and compare the results with and without compensation in terms of frame error rate.

V. Simulation Results

For the simulation results presented here we considered an UTRA FDD uplink environment according to [9]. The desired user was a speech user (Adaptive Multirate AMR with 12.2kbps) with a convolutional code of rate 1/3. The *VehicularA* channel model was chosen which comprises 2 strong and a couple of weaker paths. The velocity was 120km/h, nevertheless a power control with a 1dB step size and 4% feedback error rate was enabled. For this service, the fractional training power β^2 and the training length L are $\beta^2 = -2.69$dB and $L = 1536$. For details cf. [9].

The interference was simulated as temporally uncorrelated noise, but with spatial correlation. It is composed out of 10 other speech users and 7 data users (streaming data with 128kbps). A data user was assumed to be 6.5dB stronger than a speech user which was taken from other simulations. In addition, 37% of the total interference power was considered spatially uncorrelated noise modelling intercell-interference as well as thermal receiver noise.

As in the previous section, the array was a 6-element ULA with $\lambda/2$-spacing and a 120^o-sectorization. For the fixed beamformer, the beams of Figure 2 are used. For all users, an angular spread of 10^o was assumed.

The angular positions of all users is depicted in Figure 6. The desired user is located at 25^o which is not in the center of a fixed beam. The data users were put on top of the side lobes of a standard beam pointing into the direction of the desired user, where we had 2 users in the directions $\{-50^o, -25^o, -5^o\}$ each and a single users at 60^o. The speech users are uniformly spread over the sector. The standard beam as well as the adjacent fixed beams are also added in Figure 6.

The results are shown in Figure 7 in terms of frame error rate (FER) for the compensated and the uncompensated case. The results for perfectly known covariance matrices are omitted since they are almost identical with the compensated curves. The concepts described in Section II are considered.

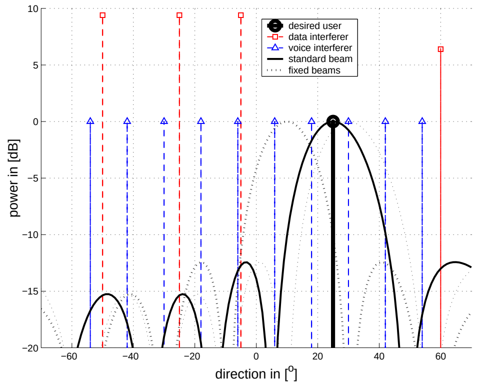

Fig. 6. Interferer Scenario.

The FER is plotted versus the C/I, where the interference power includes the data users, the speech users and the intercell interference/receiver noise. The operational point for the AMR12.2 service is at a FER of 1%.

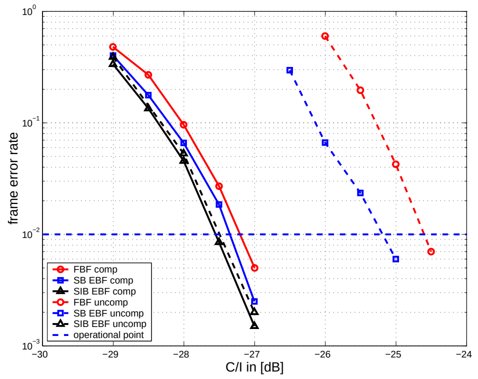

Fig. 7. Frame Error Rate for AMR Speech Service.

As expected we observe huge gains of more than 2dB for the fixed beamformer (FBF) and the signal based eigen beamformer (SB EBF) which both are based on maximizing the signal power. Here, wrong beams are generated or selected, respectively, pointing into interferer directions. The advantage for the signal and interference based eigen beamformer (SIB EBF) with compensation is minor. The correct beams are created and selected. Noisy beams might be selected, but in an implementation this effect is usually limited by limited resources for the short-term processing.

VI. Conclusion

This work has been focussed on the estimation of spatial long-term covariance matrices which are used by many beamforming techniques. In the past, these matrices often had been assumed to be perfectly known, since a long averaging process can be applied. The covariance of the desired signal is usually based on the output of the channel estimation, whereas the interference and noise covariance is approximated by the covariance of the totally received signal.

After reviewing three WCDMA beamforming concepts, namely the fixed beamformer, the signal based eigen beamformer and the signal and interference based eigen beamformer, we have studied the systematic error occurring when utilizing the estimates without any further processing. Both covariance matrix estimates contain a component of the other one, even if perfect long-term averaging is assumed.

A very simple compensation scheme has been proposed which completely removes this systematic error. We have shown, that the compensation for the covariance matrices in some cases can be replaced by simpler schemes, e.g. by compensating for the eigenvalues.

The derivation for the flat case has been extended to the frequency selective case using the standard WCDMA assumptions.

Simulation results have demonstrated, that applying the compensation almost yields perfect estimates of the covariance matrices. Not using the compensation causes huge degradations, especially for beamforming concepts based on maximizing the signal power instead of maximizing the SINR.

In any case, we need an estimate of the interference and noise covariance, either for compensating the signal covariance or for the more adaptive SINR based methods.

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