Spatial Long-Term Variations in Urban, Rural and Indoor Environments

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Abstract

A widely used property of the mobile channel is the assumption, that the spatial structure is constant over fairly a long time. In the present work we try to quantify this feature by help of a measurement campaign conducted in Vienna November 2001. We review the definition of the $F$-eigen-ratio which is a measure for the discrepancy between two covariance matrices. In urban, rural and indoor scenarios, we present results in terms of cumulative distribution functions of the $F$-eigen-ratio for different time spaces $\Delta t$, i.e. for covariance matrix pairs which are valid for time instances $t_0 - \Delta t$ and $t_0$, respectively.
In addition, the \textit{spatial long-term time constant} and the \textit{WSS quality} are defined. It turns out, that the spatial properties can be viewed constant for more than 100 doppler cycles in the urban and rural environment, but less than 20 doppler cycles in the indoor environment. The latter result makes the WSS assumption questionable for indoor environments, at least in the spatial sense.

1 Introduction

Almost all beamforming methods proposed for mobile communication systems [1] are based on the assumption, that the spatial properties of the channel change very slowly. This is strongly related to the wide sense stationarity assumption [2] which says that second order statistics can be viewed constant over certain time intervals.
Whereas a lot of algorithms apply this assumption inherently, [3] explicitly distinguishes between short-term and long-term properties. The first account for the changing interference situation ("fast fading", small-scale effects) and the latter expresses the current environment in terms of delays, doppler frequencies, direction of arrival/departure, average power ("slow-fading", large-scale effects) [4].
The spatial properties are often considered as spatial covariance matrices [5]. This document tries to quantify how fast the covariances and herewith the spatial structure vary.
Two questions should be answered which have dramatic impact on implementing beamforming algorithms. The first is about the update intervals of the spatial processing. The longer the spatial properties are constant, the less frequently we have to repeat the spatial calculations. In addition we could allow for more complex and more sophisticated algorithms. A related aspect is the long-term feedback of spatial information, which is required for some downlink beamforming strategies [6]. The second question aims at the averaging interval. Long averaging yields more stable results which make the spatial processing more precise.
The investigations are based on a measurement campaign conducted November 2001 in Vienna.
In [7] we already discussed the evaluation strategies, introduced the \textit{F-eigen-ratio} and illustrated the functioning by means of a single scenario. The F-eigen-ratio performance was verified with directional results which were obtained by SAGE evaluations in [8]. The present work can be understood as a continuation of both studies, where we now apply the discussed techniques to a large number of scenarios in an urban, a rural and an indoor environment. We will also take into account the spatial eigen properties of the covariance ensemble which we presented in [9]. We start with the most important features of the measurement equipment and the environments. In section IV we review the definition and the properties of the F-eigen-ratio. It is applied to the measurement data in section V, the results are discussed in terms of cumulative distribution functions and the \textit{spatial long-term time constant} is defined. In section VI we give some interpretation, draw the connection to the WSS assumption and define the \textit{WSS quality}. Section VII concludes this work.

2 Measurement Setup

All measurements were performed with the MIMO capable wideband vector channel sounder RUSK-ATM, manufactured by MEDAV, Germany [10]. The sounder was specifically adapted to operate at a center frequency of 2GHz with an output power of 2Watt. The measurement bandwidth was 120MHz. At the mobile side a circular 15-element array of monopoles [11] was used whereas the base station was equipped with a linear eight element patch array provided by T-Nova, Germany. With above arrangement, consecutive sets of $15 \times 8$ transfer functions, cross-multiplexed in time, were measured every 21.5ms. In addition the sixteen port of the transmit multiplexer was terminated with 50$\Omega$ providing an additional noise measurement each snapshot which is used for noise cancellation. A closer description on the equipment and on the investigated environments is given in [12]. We will summarize the major aspects of the environments in the next section.

3 Measurement Environments

The measurement data used for this paper was conducted during a measurement campaign in Vienna 2001. Main attention was drawn on measuring representative areas for mobile communication systems. For this paper the following three environments were chosen:

3.1 Urban area

Measurements were performed in downtown Vienna near the University of Technology. The receiver was placed on top of one of the highest buildings in the surrounding at a height of about 30m. The transmitter was moved on the streets within the coverage area of the equipment at speeds of about 3kmh.

3.2 Rural area

Measurements were taken at a small village near Vienna with the receiver at a height of about 20m mounted on a lift. The surrounding covers small one-family houses with a maximum height of about 10m and open places. Therefore a lot of LOS cases can be assumed as well as strong diffraction over roof-tops. The transmitter was moved with about 6kmh.
3.3 Indoor environment

In addition to the outdoor measurements the office facilities of FTW have been measured, too. This is a modern ferroconcrete office building where the receiver was placed on the aisle and the transmitter was moved through several offices. Transmitter and receiver were placed on the same floor for all measurements.

4 F-eigen-ratio

In [7] we already introduced the F-eigen-ratio. It is a measure describing the discrepancy between two covariances. The name emphasizes the fact, that the eigen structure of the matrices is considered [3].

Translated to the spatial long-term variations, we measure a spatial covariance matrix \( \mathbf{R}(t_0 - \Delta t) \) at time \( t_0 - \Delta t \) and apply this matrix at time \( t_0 \), where the spatial structure might have changed described by a covariance matrix \( \mathbf{R}(t_0) \). The F-eigen-ratio accounts for the resulting SNR loss.

We will now briefly recall the F-eigen-ratio definition. The considered covariance matrices are members of the complex space \( \mathbb{C}^{K_a \times K_a} \) where \( K_a \) is the number of antennas. We start with the eigenvalue decompositions

\[
\mathbf{R}(t_0 - \Delta t) = \hat{\mathbf{W}} \cdot \hat{\mathbf{A}} \cdot \hat{\mathbf{W}}^H \; \; ; \; \; \mathbf{R}(t_0) = \mathbf{W} \cdot \mathbf{A} \cdot \mathbf{W}^H
\]

(1)

where \( \hat{\mathbf{A}}, \mathbf{A} \in \mathbb{C}^{K_a \times K_a} \) are diagonals with the eigenvalues of \( \mathbf{R}(t_0 - \Delta t) \), \( \mathbf{R}(t_0) \) as entries, and the columns of \( \hat{\mathbf{W}}, \mathbf{W} \in \mathbb{C}^{K_a \times K_a} \) are the corresponding eigenvectors. The hat stresses the outdated nature of the eigenvectors \( \hat{\mathbf{W}} \) and eigenvalues \( \hat{\mathbf{A}} \), whereas \( \mathbf{W} \) and \( \mathbf{A} \) are the correct values, which are assumed to be unavailable.

Furthermore, we introduce the reduced versions \( \hat{\mathbf{W}}_F, \mathbf{W}_F \in \mathbb{C}^{K_a \times F} \) of the matrices \( \hat{\mathbf{W}}, \mathbf{W} \), which contain the eigenvectors corresponding to the \( F \) largest eigenvalues of the covariance matrices \( \mathbf{R}(t_0 - \Delta t) \) and \( \mathbf{R}(t_0) \), respectively. Then, the F-eigen-ratio is defined as

\[
q_{\text{eigen}}^{(F)}(\Delta t) = \frac{\text{tr} \left\{ \hat{\mathbf{W}}_F^H \cdot \mathbf{R}(t_0) \cdot \mathbf{W}_F \right\} }{\text{tr} \left\{ \hat{\mathbf{W}}_F^H \cdot \mathbf{R}(t_0 - \Delta t) \cdot \mathbf{W}_F \right\} }
\]

(2)

with the properties \( 0 \leq q_{\text{eigen}}^{(F)}(\Delta t) \leq 1 \), \( q_{\text{eigen}}^{(F=K_a)}(\Delta t) = 1 \forall \Delta t \) and \( q_{\text{eigen}}^{(F)}(0) = 1 \forall F \).

In other words, the F-eigen-ratio expresses the loss due to the application of outdated antenna weights \( \hat{\mathbf{W}}_F \) instead of the correct weights \( \mathbf{W}_F \).

In the line-of-sight case, \( q_{\text{eigen}}^{(F=1)} \) matches beam pattern values, where the azimuth axis is transformed to \( \Delta t \) values. This is demonstrated in [8] for one LOS scenario.

With larger \( F \)-values, \( q_{\text{eigen}}^{(F)} \) typically decreases, since it is more likely to hit energy carrying dimensions in the current signal space with the outdated weight vectors \( \hat{\mathbf{W}}_F \). However, the columns in \( \hat{\mathbf{W}}_F \) do not decorrelate the channel, i.e. the off-diagonal values of \( \hat{\mathbf{W}}_F^H \cdot \mathbf{R}(t_0) \cdot \mathbf{W}_F \) are non-zero. This effect is not captured by the F-eigen-ratio.

5 Results

From all measurements we extracted spatial covariance matrices as described in [7], which will be reviewed in the sequel.
5.1 Extraction of the Covariance Matrices

From the measured impulse responses a 5MHz band was used only, which is a typical bandwidth for 3rd generation mobile communication [13]. The spatial covariance matrix was set up by incoherently averaging over all delay values, all transmit antennas and a time interval $\Delta t_{\text{avg}}$. In addition, a separately measured noise matrix was subtracted and the matrices were corrected by a calibration matrix [14]. A covariance matrix was initiated each $\Delta t_{\text{new}}$ seconds. The resulting set of investigated matrices is exactly the same as used in [9] for the spatial evaluation. Special attention was paid to the choice of the parameters $\Delta t_{\text{avg}}$. On one hand, it should be large enough so that all short-term effects are eliminated by averaging. Otherwise, the rank of the matrices would decrease. On the other hand, a too large value would violate the WSS assumption already within a single covariance pretending a higher rank than the true one.

The chosen values of $\Delta t_{\text{avg}} = 4.3$sec for the urban and rural and $\Delta t_{\text{avg}} = 2.15$sec for the indoor environment resulted in relatively smooth F-eigen-ratio curves over time [7]. This suggests that the averaging was long enough to achieve stable matrices. On the other hand, a result of [9] is that the rank of the matrices is even smaller than expected, so that the choice of $\Delta t_{\text{avg}}$ seems to be reasonable.

The parameter $\Delta t_{\text{new}}$ was set to a slightly smaller value than $\Delta t_{\text{avg}}$ to establish a small temporal overlapping between the matrices. For the urban and rural case we used $\Delta t_{\text{new}} = 3.2$sec and for the indoor case $\Delta t_{\text{new}} = 1.6$sec. The values of $\Delta t$ for the following investigations are multiples of $\Delta t_{\text{new}}$.

5.2 Common Results

For a certain value of $\Delta t$ we considered all occurring pairs $\{\mathbf{R}(t_0); \mathbf{R}(t_0 + \Delta t)\}$. For each pair, the F-eigen-ratio $q_{\text{eigen}}^{(F)}(\Delta t)$ was evaluated using $F = 1$ as well as $F = 2$. Each curve in the Figures 1, 2 and 3 depicts the cumulative distribution function (CDF) of the F-eigen-ratio $q_{\text{eigen}}^{(F)}(\Delta t)$ for a single value of $\Delta t$. The left plots are the results for $F = 1$ and the right plots for $F = 2$.

First, we will discuss some common properties of all plots:

- For the smallest temporal spacing $\Delta t = \Delta t_{\text{new}}$, the curves (cycles) are very close to the step function. This a posteriori verifies the reasonable choice of $\Delta t_{\text{avg}}$.  

![Figure 1: F-eigen-ratio vs. Time for the Urban Environment ($\Delta t_{\text{avg}} = 4.3s \alpha; \Delta t_{\text{new}} = 3.2sec$)](image-url)
Figure 2: F-eigen-ratio vs. Time for the Rural Environment ($\Delta t_{avg} = 4.3\, sec; \Delta t_{new} = 3.2\, sec$)

- The smoothness of the curves decreases for higher values of $\Delta t$, since less covariance pairs and herewith less samples of $q^{(F)}_{\text{eigen}}(\Delta t)$ are available.

- The $F = 2$ curves are always above the $F = 1$ curves. If we use more outdated eigenvectors, it is more likely to hit energy carrying dimensions in the current signal space.

- In each plot we added one curve for a very high value of $\Delta t$. However, even there $q^{(F)}_{\text{eigen}}(\Delta t)$ seems to be relatively low in a lot of cases. For instance static scatterers or radial movement lead to "quasi-time-invariance". For such scenarios, $q^{(F)}_{\text{eigen}}(\Delta t)$ will be very low.

- With increasing $\Delta t$ values the curves shift downwards. Considering a single scenario, $q^{(F)}_{\text{eigen}}(\Delta t)$ is not necessarily monotone in time (cf. [8]) due to e.g. side lobes of the beam patterns etc.

Before assessing the individual environments, we will define the spatial long-term time constant $\tau_{LT}$:

**Definition:** The spatial long-term time constant $\tau_{LT}$ is the time difference $\Delta t$ for which the $(F = 1)$-eigen-ratio $q^{(F=1)}_{\text{eigen}}(\Delta t)$ is less than 1dB in 90% of all cases.

This time constant corresponds to the CDF curve crossing the point (1dB; 90%). This definition seems to be reasonable, albeit it is initially arbitrary. Even if we used a spatial covariance matrix for this period we would have a maximal loss of 1dB at the end of the period only (in 90% of the cases). So, the total performance loss will be much lower than 1dB. Other definitions of the long-term time constant would change the following results quantitatively, but not qualitatively. Note, that for a similar definition using $F > 1$, the constant $\tau_{LT}$ would be larger. We will now proceed with the discussion of the individual environments.

5.3 Individual Results

Figure 1 shows the results for the urban environment. With increasing $\Delta t$ values the CDFs drop down very moderately. The (1dB; 90%) point is crossed by the $\Delta t = \tau_{LT} = 22.6\, sec$ curve. The combination of over-roof-top diffraction with local scatterers and the relatively broad beam widths leads to a rather small dependence on time.
A result of [9] is, that in this environment 1-2 spatial dimensions are available for the employed array. That is, the $F = 2$ curve might be of interest as well. As long as the applied spatial algorithm is capable to handle 2 dimensions, the loss for $\Delta t = 22.6$sec is much smaller ($\approx 0.6$dB at 90\% ) or, equivalently, we could allow for a longer $\Delta t$. For the signal processing way of thinking, the range of 20sec seems to be very high, but we should keep in mind, that the velocity was rather small (ca. 3kmh).

As already mentioned, the velocity was higher in the rural environment (ca. 6kmh). However, this is not the only reason that the CDFs drop more rapidly in Figure 2. The small distance between transmitter and receiver (ca. 50m-500m) causes high angular velocities. And since we have almost exclusively line-of-sight scenarios we quickly run out of the beam, at least in the case of tangential movement (cf. [8]). The long-term time constant reads as $\tau_{LT} = 9.7$sec.

The evaluations in [9] show that there is nearly always a single spatial dimension, so the $F = 2$ plot is added for the sake of completeness only and does not contain additional information.

In the indoor environment we obviously did not completely average out the short-term effects due to the smaller interval $\Delta t_{avg}$. We already have a significant $F$-eigen-ratio for the smallest step size of $\Delta t = 1.8$sec. This setting was necessary, since we expected the long-term variations to be faster than in the other environments. Figure 3 verifies this supposition. The long-term time constant is only $\tau_{LT} = 4.8$sec. Even for short movements in the range of some meters we could have a different environment, e.g. another room, despite the low velocity of $\approx 1.5$kmh. An interesting aspect is, that the CDFs do not significantly drop further down for $\Delta t > 4.8$sec. However, [9] suggests that usually 2 or more spatial dimensions are available in this environment.

![CDF plots for $F=1$ and $F=2$](image)

Figure 3: $F$-eigen-ratio vs. Time for the Indoor Environment ($\Delta t_{avg} = 2.15$s $\alpha$; $\Delta t_{new} = 1.6$s $\alpha$)

Hence, depending on the implemented algorithms, we should have a look at the $F = 2$ plot as well. It seems, that we can collect almost full power with 2 outdated dimensions even for larger time spacings. But note, that we would not project on the eigen dimensions, i.e. the outdated weights $W_F$ would not decorrelate the channel. As a consequence, long-term diagonalization for MIMO systems [15] might not be possible, even if the $F$-eigen-ratio is small.
6 Interpretation

The results presented in the previous section are valid for the recorded environments, i.e. for specific velocities, specific antenna locations etc. A generalization of the results might be very dangerous, at least in a quantitative manner. In the sequel, we will give some considerations which should help to draw generalized qualitative conclusions.

6.1 Wide Sense Stationarity

First we try to assess, to what extend the WSS assumption is fulfilled for our setup. To this end, we compare the long-term time constant \( \tau_{LT} \) derived in the previous section with the coherence time of the channel \( \tau_{coh} \).

The most common definition of the coherence time is the duration of one cycle of the maximum doppler frequency. For the three environments, the velocities are 3kmh, 6kmh and 1.5kmh which translates to doppler frequencies of 5.6Hz, 11.1Hz and 2.6Hz, respectively, for a carrier frequency of 2GHz. The coherence time is the reciprocal of the doppler frequency, i.e. 180ms, 90ms and 360ms for the urban, rural and indoor environment. This pretty well matches the temporal measurement results in [9].

We define the spatial WSS quality \( Q_{WSS} \) as the quotient

\[
Q_{WSS} = \frac{\tau_{LT}}{\tau_{coh}}
\]

which is a number expressing how many multiples of the coherence time the spatial properties can be considered constant.

Table 1 summarizes the results for the WSS quality. In the urban and rural environment, the spatial long-term properties remain constant for roughly \( Q_{WSS} = 100 \) doppler cycles (fading periods). However, the value \( Q_{WSS} = 13 \) for the indoor environment is significantly smaller.

<table>
<thead>
<tr>
<th>environment</th>
<th>urban</th>
<th>rural</th>
<th>indoor</th>
</tr>
</thead>
<tbody>
<tr>
<td>long-term time constant ( \tau_{LT} )</td>
<td>22.6sec</td>
<td>9.7sec</td>
<td>4.8sec</td>
</tr>
<tr>
<td>coherence time ( \tau_{coh} )</td>
<td>180ms</td>
<td>90ms</td>
<td>360ms</td>
</tr>
<tr>
<td>WSS Quality ( Q_{WSS} )</td>
<td>126</td>
<td>108</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 1: Long-term Time constant, Coherence Time and WSS Quality

6.2 Transition to higher Velocities

In mobile communications we are often interested in higher velocities than investigated here. To obtain reliable results for these cases, we would have to repeat the measurements. This might be very exhaustive since the equipment must be mounted e.g. on a car. In addition, high velocities cause electrical problems as we might not be able to track the high doppler frequencies. Therefore, we will now argue how we can transfer the presented results to higher velocities.

As a first approximation we scale down both the long-term time constant and the coherence time linearly with the velocity. This would not affect the WSS quality (3).

While scaling the coherence time seems to be reasonable this is probably not true for the long-term time constant. First, in some scenarios we have almost no dependence on time at all, e.g. in the case of radial movement. These scenarios are not sensible to higher velocities which would distort the statistical CDFs towards lower values. Second, high velocities are not possible in the investigated
environments. Environments which allow for high velocities usually feature a smaller dependence on the location. For instance on a highway, the spatial structure will be constant over possibly tens of meters, in contrast to offices where the spatial structure can completely change within a few meters.

These two reasons suggest that the spatial long-term time constant decreases weaker than linearly with the velocity. Hence, the WSS quality will increase with higher velocities. We will not dare to give any quantitative guess at this point, but we consider the values in Table 1 as lower bounds and keep in mind that they will be larger for higher velocities.

6.3 Consequences

The significantly small $Q_{WSS}$ value in the indoor environment indicates, that the WSS assumption is poorly fulfilled there. In other words, the difference between the long-term scale and the short-term scale is minor. Hence, we might not be able to set up reliable and stable covariance matrices. Beamforming in the original sense might be critical.

However, the velocity typically is very small leading to a high coherence time. As a consequence, we can allow for high coherent averaging intervals (long training sequences) achieving a high accuracy for the instantaneous impulse response estimates.

In the urban and rural environment the difference between the short-term and long-term scale is very large. This enables long non-coherent averaging intervals in order to determine stable second order statistics in space, e.g. covariance matrices. The update intervals for the spatial processing, e.g. eigenvalue decompositions [3], direction finding [1] etc., can be chosen rather long decreasing the average complexity. In addition, there is long time available for the feedback of spatial information in downlink beamforming schemes [6]. For instance, when scaling down the spatial long-term time constant with the velocity (this was stated to be the worst case in the previous section), it is still in the range of 0.5sec at 120kmh which is sufficient for most communication systems such as UMTS [13].

7 Conclusion

We investigated the long-term stability of spatial covariance matrices by help of a measurement campaign conducted in Vienna November 2001. After a brief description of the equipment and the environments, we reviewed the F-eigen-ratio which expresses the discrepancy between two covariance matrices.

The F-eigen-ratio was evaluated for all available pairs of covariances which are valid for time instances separated by a certain time spacing $\Delta t$. The results were given in terms of CDFs for each considered environment, for different values of $\Delta t$ and for $F = 1$ and $F = 2$.

We defined the spatial long-term time constant $\tau_{LT}$ as the spacing $\Delta t$ where the F-eigen-ratio is less than 1dB in 90% of all cases. We assume that spatial covariance matrices can be viewed constant over $\tau_{LT}$. The constant was found to be 22.6sec, 9.7sec and 4.8sec in the urban, rural and indoor environment, respectively.

In addition, we defined the spatial WSS quality $Q_{WSS}$ which is the quotient of the long-term time constant and the coherence time of the channel. It expresses, to what extend the WSS assumption is fulfilled in the spatial sense. The result was, that spatial second order statistics, such as the covariances, are constant over more than 100 cycles of the maximal doppler frequency for the urban and rural environment, but only in the range of 10 doppler cycles for the indoor environment.

Finally, we discussed the influence of the velocity as well as consequences on beamforming algorithms.
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